Int. J. Heat Mass Transfer. Vol. 21, pp. 537–542
© Pergamon Press Ltd. 1978. Printed in Great Britain

0017-9310/78/0501-0537 $02.00/0

HEAT-TRANSFER MEASUREMENTS OF EVAPORATING LIQUID DROPLETS

M. C. YUEN and L. W. CHEN
Department of Mechanical Engineering and Astronautical Sciences, Northwestern University,
Evanston, IL 60201, U.S.A.

(*Received* 7 *January* 1977 *and in revised form* 14 *September* 1977)

Abstract—The heat transfer to simulated (porous sphere) water and methanol droplets were measured in an atmospheric vertical hot air tunnel. The experiments were limited to a Reynolds number range of 200–2000, a free stream air temperature range of 150–960°C and a velocity range of 2.1–11.4 m/s. The experimental data can best be correlated by

$$Nu_f(1+B) = 2+0.6Re_M^{1/2}Pr_f^{1/3}$$

where properties are evaluated at film conditions except for the density in the Reynolds number which is the free stream density. Thus the data show that at higher temperatures, evaporation reduces heat-transfer rates directly by a factor of $(1+B)$. Indirectly evaporation affects heat-transfer rates through the changes in both the compositions and the temperatures of the surrounding gaseous medium.

NOMENCLATURE

A,	area;
B,	$(h_s - h_d)/L$, mass-transfer number;
C_p,	at specific heat of constant pressure;
D,	mass-diffusion coefficient;
d,	diameter of porous sphere;
h,	heat-transfer coefficient;
h,	enthalpy per unit mass;
k,	thermal conductivity;
L,	latent heat of vaporization per unit mass;
$\dot{m}$,	mass flux;
Nu,	$= hd/k$, Nusselt number;
p,	constant;
ΔP,	dynamic pressure;
Pr,	$= C_p\mu/k$, Prandtl number;
Q,	$= Q_C + Q_R$, total heat transfer;
Q_1,	heat conduction through tube wall;
Q_2,	heat conduction through liquid in the tube;
Re,	$= \rho ud/\mu$, Reynolds number;
Re_M,	$= \rho_s ud/\mu_f$;
Sc,	$= \mu/\rho D$, Schmidt number;
T,	temperature;
ΔT,	$= T_s - T_d$;
u,	velocity.

Greek symbols

μ,	viscosity coefficient;
ρ,	density.

Subscripts

a,	air;
c,	convection;
d,	droplet surface;
f,	film;
l,	liquid;
R,	radiation;
r,	reference condition using 1/3 rule;
s,	free stream;
v,	vapor;
2,	entrance to sphere.

INTRODUCTION

IN THE analysis of engineering problems involving liquid spray, knowledge of the behavior of an evaporating liquid droplet is essential for the understanding and the prediction of the performance of the spray as part of an engineering system. Liquid spray occurs, for example, in problems of combustion, fire extinguishment, nuclear reactor safety, cooling towers, etc.

When a liquid droplet is suddenly exposed to a hot environment, initially the temperature of the droplet rises rapidly. In local thermodynamic equilibrium the partial pressure of the vapor at the surface is a function of the surface temperature of the droplet. The vapor concentration at the surface is a balance of production on the one hand and diffusion and convection on the other hand. Concurrently, the production of vapor requires an external heat source to provide enough energy to overcome the heat of vaporization. This cannot come from the droplet alone. The net result of the interactions of heat transfer, vapor production, vapor concentration, droplet temperature and free stream temperature is that for a constant free stream pressure the droplet temperature (for each liquid) is only a function of the free stream temperature. This is true for a droplet in quasi-steady state. Experimental measurement shows there is a slight dependence of droplet temperature on Reynolds numbers which can be neglected for most cases. Thus, in quasi-steady state, the heat transfer to the sphere results only in droplet vaporization. Under this condition, heat and mass transfer are energetically equivalent.

The effect of evaporation is therefore two fold. Firstly, because of evaporation, the temperature and the composition surrounding the droplet are different from the free stream values; secondly, mass efflux from evaporation may affect the flow field in the vicinity of the droplet. Both of these processes will affect the heat and mass transfer to the droplet.

For quantitative work on behavior of droplet in quasi-steady state, the two important measurements are the drag coefficient and heat-transfer rate to the droplet. The drag coefficient measurements of evaporating droplets by Yuen and Chen [1] together with those of Eisenklam, Armachalam and Weston [2] cover the entire range of Reynolds numbers from 1 to 2000 and temperature range from 25 to 1000°C. The experimental data agree well with the "standard" drag curve provided the characteristic density is the free stream density and the characteristic viscosity coefficient, $\mu_r(T_r)$, is evaluated according to the 1/3 rule [3]. The present correlation is insensitive to the mass-transfer number suggesting that mass efflux has little effect on drag of evaporating droplets.

There is extensive literature, both theoretical and experimental, on the heat- and mass-transfer measurements of various liquid droplets in stagnant environment or in forced convection. For low Reynolds numbers, earlier work in this field was reviewed and summarized by Fuchs [4]. Of the earlier work, the most important measurements were those by Frössling [5], and Ranz and Marshall [6]. The empirical formula which best fitted the experimental data of [5, 6] is

$$Nu = 2+0.6Re^{1/2}Pr^{1/3}. \tag{1}$$

In equation (1), the thermodynamic and transport properties of the dimensionless numbers are evaluated at the film temperature and the film composition. At low temperature, the effect of vapor concentration can be neglected but this is not true at higher temperatures. Although their data is limited to Reynolds numbers below 200, Ranz and Marshall have shown that equation (1) correlates well with other heat-transfer data of sphere to a Reynolds number of 40000. Hsu, Sato and Sage [7] obtained similar correlation for evaporating heptane droplets.

Equation (1), however, does not agree with the widely quoted correlation of McAdams [8] for forced convective heat transfer to a sphere. It is

$$Nu = 0.37Re_M^{0.6}. \tag{2}$$

In 1965, Rowe, Claxton and Lewis [9] reviewed earlier work of heat and mass transfer for a single sphere with particular emphasis in the range of Reynolds numbers less than 2000. They also presented their data and found that the exponent for Reynolds number depended on the range of Reynolds numbers. For $Re < 2000$, the exponent 1/2 is a good approximation. Their experimental heat-transfer data in air in the range of Re from 60 to 1800 give the following correlation:

$$Nu = 2+0.69Re^{1/2}Pr^{1/3}. \tag{3}$$

In equation (3), the coefficient in the term of $Re^{1/2}Pr^{1/3}$ is 0.69 instead of 0.6 in equation (1). Thus equation (3) predicts a higher heat-transfer rate for the same Reynolds number. One possibility is that for relatively large spheres (1.25–3.81 cm dia) and low Reynolds numbers (60–1800), the effect of free convection is not negligible.

There is still more recent work such as that by Raithby and Eckert [10], where, the Reynolds numbers ranged from about 3.6×10^3 to 5.2×10^4. From Re between 3×10^3 to 10^4, their correlation agreed quite well with equation (1) but below 10^3 they deviated significantly from equation (1). Although measurements were carefully made, the empirical correlation could not be automatically extended to lower Reynolds number range. Therefore it does not necessarily invalidate equation (1).

For heat and mass transfer of liquid droplets, the range of Reynolds numbers which is of interest in almost all cases is generally well below 2000. Thus we believe that the data of Ranz and Marshall represent the best in literature for evaporating liquid droplets at low temperature in this range of Reynolds numbers. As such, equation (1) serves as the standard curve for all other data.

For the measurement of rates of evaporation and heat transfer of larger droplets, Ingebo [11] used a wetted cork sphere (3.44 mm radius) soaked in a number of organic liquids to simulate the droplets, and found the correlation in a slightly different form. Representative data of [11] together with equation (1) are shown in Fig. 1. Only the data of water and methanol are shown. They cover a temperature range of up to 400°C. Figure 1 shows that the measured heat-transfer rate of [11] is too high.

Recently, Eisenklam, Armachalam and Weston [2] reported that at higher temperatures, the effect of mass transfer on heat transfer of liquid droplets can not be neglected. They determined the evaporation rate photographically by measuring the size of a free falling droplet in a hot air column. For purpose of comparison we have shown in Fig. 1 the water and methanol data of [2]. The data indicate a reduction in heat transfer for the same Reynolds number as the ambient temperature increases. However the low

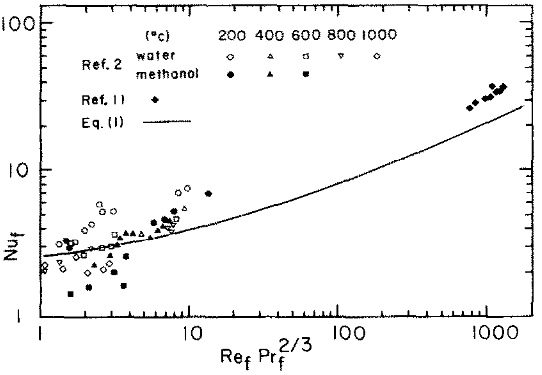

FIG. 1. Comparison of heat-transfer data of evaporating liquid droplets with standard curve.

temperature data (200°C) are much higher than equation (1). This indicates that the experimentally measured heat-transfer rates are too high.

Figure 1 and other published data indicate that heat- and mass-transfer rates at low temperature are accurately known. This is not true in the higher temperature range. The purpose of the present experiment is to attempt to measure as accurately as possible the heat-transfer rate of various liquid droplets at higher temperatures. Hopefully, this will provide the confidence needed in the data at the high temperature range that we now enjoy at the low temperature range.

In the following section we shall discuss the experiment which includes the method of measurement, experimental set-up and experimental procedure. Finally we shall present the experimental data and discuss the correlation of the experimental data.

EXPERIMENT

In the present experiment we shall simulate a droplet by using a porous sphere soaked in the liquid and to regulate the supply of liquid so that the surface is always wetted. At steady state, the rate of liquid flow to cover the entire surface is the rate of evaporation.

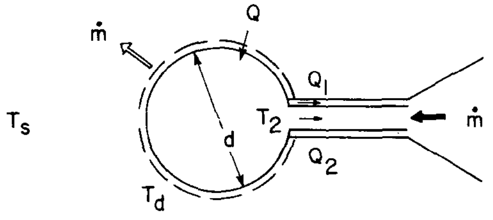

FIG. 2. Schematic diagram of porous sphere.

To study the steady state heat-transfer rate, we shall consider the mass and energy flux across the control volume as shown in Fig. 2. The equation for the conservation of energy is

$$\dot{m}h_l + Q = \dot{m}h_v + Q_1 + Q_2 \tag{4}$$

where $\dot{m}$ is the mass flux, h_l and h_v are the enthalpies of liquid and vapor, Q is the heat transfer to the sphere, Q_1 and Q_2 are the heat losses through the tube wall and liquid in the tube by heat conduction. In this case, order of magnitude calculation shows that Q_1 and Q_2 are small compared with Q. Neglecting Q_1 and Q_2, equation (4) becomes

$$Q = \dot{m}L(T_d) + \dot{m}[h_l(T_d) - h_l(T_2)] \tag{5}$$

where L is the latent heat of vaporization.

For a droplet in a wind tunnel, the radiative heat transfer to the droplet comes mainly from the hot walls and the heater, and the contribution from the gas is negligible. The convective heat-transfer Q_c is therefore

$$Q_c = Q - Q_R. \tag{6}$$

The effect of natural convection in this case has been estimated to be negligible, i.e.

$$\frac{Q_{\text{free convection}}}{Q_{\text{forced convection}}} \simeq 0(0.01)$$

for the case of lowest Reynolds number which is about 200. Thus, Q_c in equation (6) represents only heat transfer from forced convection.

A schematic of the experimental set-up is shown in Fig. 3. A detailed description is presented in [1] and [12] and will not be repeated here. Instead the operational characteristic will be summarized below.

The test section of the vertical wind tunnel is 35.6 cm long with a 7.6 × 7.6 cm cross-section. Two sides of the test section are made of Vycor glass for viewing purposes. The wind tunnel can be operated up to a temperature of 1000°C and flow velocity up to 20 m/s.

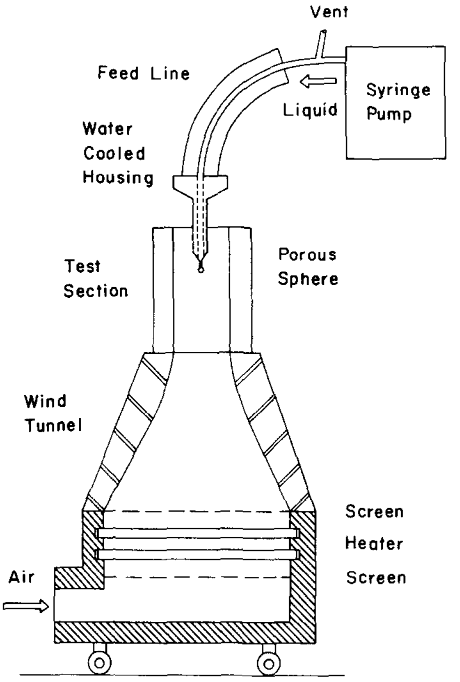

FIG. 3. Sketch of the experimental set-up.

The diameter of the sintered bronze porous sphere is 6.35 mm and the stainless steel tube which feeds liquid to the sphere has an OD of 1.3 mm and an ID of 0.8 mm. There is a thermocouple located inside the tube at the entrance of the porous sphere in order to measure the liquid temperature T_2.

The liquid feed line from the syringe pump to the porous sphere is protected from the hot gas by a water cooled brass housing and a brass elbow. The stainless steel tubing of the porous sphere is soldered to the conical tip of the brass housing. The length of the exposed tube varies from 0.3 to 0.95 cm. If this length is too short, it will interfere with the heat transfer to the sphere; if it is too long, the liquid inside the tube gets too hot. We have found that in the range of 0.3–0.95 cm, the heat-transfer rate remained the same. We have also found that it was important to eliminate any trapped air bubble in the feed line and in the porous sphere. This was accomplished by immersing the porous sphere in the liquid and reversing the flow of the liquid so that air bubbles were forced out through the vent.

Before the start of the experiment, the liquid feed line is purged of any air bubbles and the porous sphere is completely immersed in the test liquid. When it is ready to position the porous sphere into the test section, the syringe pump is turned on to insure that the porous sphere is completely covered with liquid. After the porous sphere is positioned, the flow rate of the test liquid can be controlled by varying the gear ratio and the voltage to the DC motor. When the flow rate is too high, excess liquid will form a drop at the forward stagnation point, thus increasing the droplet size. When the flow rate is too low, there will not be enough liquid to cover the entire surface of the sphere. Patches of dry spots will appear. In some cases, the entire surface becomes dry. The rate of evaporation for a specific free stream temperature and velocity is the mass flow rate ($\dot{m}$) of the liquid when there is just enough liquid to completely wet and cover the porous sphere. Because the flow rate is small, it takes time to observe whether steady state is reached. The rule of thumb is that if no change occurs for a period of about three minutes, i.e. no fluid accumulation or appearance of dry spot, then we assume steady state is reached. In general, after the wind tunnel has reached steady state, it takes on the average about 15–20 min to determine the final steady state evaporation rate. Once the volume flow rate is measured, the temperature and dynamic pressure of the hot gas (T_s and ΔP) *and the temperature of the liquid at entrance to the sphere* (T_2) are also recorded.

The reduction of the data is as follows: The mass flux $\dot{m}$ is the product of the volume flow rate and the liquid density at room temperature. Using equation (5), one can then calculate the local heat transfer Q. The radiative heat transfer Q_R between the liquid droplet and the surrounding was calculated from the measured wall and heater temperatures. The detail of the calculation is shown in [12] and will not be repeated here. The convective heat transfer Q_c then is the difference between Q and Q_R. The heat-transfer coefficient h is computed from the equation

$$Q_c = hA\Delta T \qquad (7)$$

where $\Delta T = T_s - T_d$, and A is the surface area of the sphere minus the cross-sectional area of the tube.

RESULTS AND DISCUSSIONS

The present measurements of heat-transfer rates of liquid droplets are limited to water and methanol in the temperature range of 150–960°C and the velocity range of 2.5–11.4 m/s. A total of 60 data points were recorded.

From dimensional analysis [13], one can show that

$$Nu = Nu(Re, Pr, Sc, B, T_d/T_s). \qquad (8)$$

Ranz and Marshall have shown that at low temperatures, the Sc number in mass transfer plays the same role as the Pr number in heat transfer. In particular, the Sc number does not appear in the correlation of heat-transfer data. The effect of T_d/T_s is taken into account by appropriately defining the reference states to evaluate the thermodynamic and the transport properties. Following [6] we shall assume in the present case that heat transfer is independent of Sc number and that the effect of T_d/T_s is represented by appropriate reference states for evaluation of properties. Therefore, we have

$$Nu = Nu(Re, Pr, B). \qquad (9)$$

Any successful correlation of experimental data should agree with equation (1) or the equivalence at low temperatures. Thus in Fig. 4 we have presented our experimental data together with equation (1). The evaluation of transport properties at film conditions including changes in compositions is shown in [12].

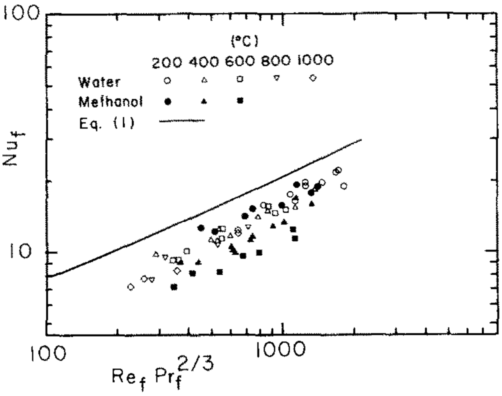

FIG. 4. Dimensionless heat-transfer rates of evaporating liquid droplets as a function of Reynolds numbers and Prandtl numbers.

All the data are grouped approximately into five temperature ranges from 200–1000°C for identification purpose. The exact temperature and other pertinent information for each of the data points are listed in [12]. As expected the data lie below the standard curve indicating the reduction in heat transfer by evaporation. Since [2] indicates the reduction in heat transfer is proportional to $1+B$, we shall assume the following correlation

$$Nu_f(1+B)^P = 2+0.6Re_f^{1/2}Pr_f^{1/3}. \qquad (10)$$

The mass-transfer number B is defined as

$$B = (h_s - h_d)/L(T_d).$$

The values of B for different T_s and liquids are listed in Table 1. The present values of B are much lower than the values of B in [2].

Table 1. Mass-transfer numbers as a function of free stream temperatures

T_s (°C)	Water	Methanol
100	0.028	0.064
200	0.064	0.14
400	0.14	0.32
600	0.23	0.49
800	0.33	
1000	0.43	

Equation (10) is a generalization of equation (1). At room temperature, B is negligible and therefore equation (10) agrees with equation (1). In equation (1), properties are evaluated at film conditions, thus the only free parameter in equation (10) is p. In Fig. 8 of [12] we have plotted the experimental data with properties evaluated at film condition and have found the best fit of the data is $p = 1.4$. The correlation is only fair. The scatter in the water data seems to be larger than the methanol data. The methanol data are systematically lower.

A further look at the data of Ranz and Marshall shows that although they evaluated properties at film temperature, the data also agree well with equation (1) even if properties are evaluated at other conditions. This is because at room temperature, the difference between free stream and droplet temperature is small. For example changing the density in Reynolds number from $\rho_a(T_f)$ to $\rho_a(T_s)$ would change the Reynolds number by 1–2% which is well within the experimental error. One can therefore generalize equation (10) to include the definition of properties as an extra degree of freedom to correlate the experimental data. In Fig. 5 we have plotted the experimental data with properties evaluated at film conditions except for density in the Reynolds number which is the free stream density. Thus the correlation that best fit the data is

$$Nu_f(1+B) = 2+0.6Re_M^{1/2}Pr_f^{1/3}. \qquad (11)$$

The definition of Re number as $\rho_a ud/\mu_f$ is reasonable. If we interpret the Re number as the ratio of inertia force to viscous force, then it is perhaps more appropriate in the case of a sphere to define the inertia force as $\rho_a u^2$ rather than $\rho_f u^2$. In fact McAdams used the same definition for Re number in his correlation of heat-transfer data of sphere. In our own work on drag of an evaporating liquid droplet [1], we have also found that it is more appropriate to use $\rho_a(T_s)$ in the definition of the Re number. However, in the case of drag measurement, empirically it is better to use 1/3 rule [3] to evaluate properties. This is not surprising since Reynolds analogy does not apply in the case of a sphere where form drag dominates at moderate Re numbers.

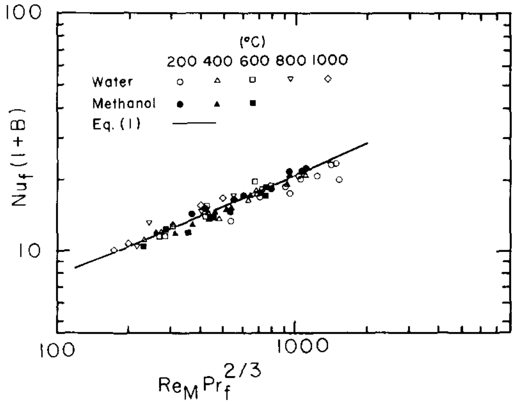

Fig. 5. Correlation of heat-transfer rates of evaporating liquid droplets in air.

CONCLUSION

The present study shows that for evaporation water and methanol droplets, the Nusselt number as a function of the Reynolds number, the Prandtl number and the mass-transfer number correlates well with equation (11). In equation (11), all properties are evaluated at the film conditions except for the density in the Reynolds number which is the free stream density. For low temperature, equation (11) reduces to the standard heat-transfer curve of equation (1). Thus the experimental data show that at higher temperatures, evaporation reduces heat-transfer rates directly by a factor of $(1+B)$. Indirectly evaporation affects heat-transfer rates through the changes in both the compositions and the temperatures of the surrounding gaseous medium.

The present study is limited to water and methanol droplets with maximum $B \simeq 0.5$ and Reynolds numbers from 200–1800. To ascertain the correctness of the correlation, further measurements should be made to higher mass-transfer numbers and lower Reynolds numbers.

Acknowledgement—This research is supported by the National Science Foundation and Northwestern University.

REFERENCES

1. M. C. Yuen and L. W. Chen, On drag of evaporating liquid droplet, *Combust. Sci. Technol.* **14**, 147–154 (1976).
2. P. Eisenklam, S. A. Armachalam and J. A. Weston, Evaporation rates and drag resistance of burning drops; in *Eleventh Symposium (International) on Combustion*, pp. 715–727 (1967).
3. G. L. Hubbard, V. E. Denny and A. F. Mills, Droplet evaporation: effects of transients and variable properties, *Int. J. Heat Mass Transfer* **18**, 1003–1008 (1975).
4. N. A. Fuchs, *Evaporation and Droplet Growth in Gaseous Media*, Translated by J. M. Pratt. Pergamon Press, Oxford (1959).
5. N. Frössling, Uber die Verdungtung Fallenden Tropfen, *Gerlands Beitr. Geophys.* **52**, 170–216 (1938).
6. W. E. Ranz and W. R. Marshall, Evaporation from drops—I, —II, *Chem. Engng Prog.* **48**, 141–146, 173–180 (1952).
7. N. T. Hsu, K. Sato and B. H. Sage, Material transfer in turbulent gas stream, *Ind. Engng Chem.* **46**, 870–876 (1954).
8. W. H. McAdams, *Heat Transmission*, 3rd edn. McGraw-Hill, New York (1953).
9. P. N. Rowe, K. T. Claxton and J. B. Lewis, Heat and mass transfer from a single sphere in an extensive flowing fluid, *Trans. Instn Chem. Engrs* **43**, T14–T31 (1965).
10. G. D. Raithby and E. R. G. Eckert, The effect of turbulence parameters and support position on the heat transfer from spheres, *Int. J. Heat Mass Transfer* **11**, 1233–1252 (1968).
11. R. D. Ingebo, Vaporization rates and heat transfer coefficients for pure liquid drops, NACA Reports, TN 2368 (1951).
12. M. C. Yuen and L. W. Chen, Heat transfer measurements of evaporating liquid droplets, NU-ME/AS Report 76-2, Department of Mechanical Engineering, Northwestern University (1976).
13. E. R. G. Eckert and R. M. Drake, *Analysis of Heat and Mass Transfer*. McGraw-Hill, New York (1972).

MESURES DE TRANSFERT THERMIQUE LORS DE L'EVAPORATION DE GOUTTES LIQUIDES

Résumé—On mesure le transfert thermique pour des gouttes simulées (sphère poreuse) d'eau et de méthanol dans une soufflerie verticale d'air chaud. Les expériences sont limitées à un domaine de nombre de Reynolds allant de 200 à 2000, à des températures d'air variant entre 150 et 960°C et à des vitesses comprises entre 2,1 m/s et 11,4 m/s. Les résultats expérimentaux sont bien représentés par la relation

$$Nu_f(1+B) = 2+0{,}6Re_M^{1/2}Pr_f^{1/3}$$

dans laquelle les grandeurs sont évaluées à la température de film, à l'exception de la masse volumique dans le nombre de Reynolds qui est celle de l'écoulement libre. Les résultats montrent ainsi qu'à des températures élevées, l'évaporation réduit les flux thermiques par un facteur (1 + B). L'évaporation affecte indirectement les flux de transfert à travers les changements de composition et de température du milieu gazeux environnant.

MESSUNG DES WÄRMEÜBERGANGS BEIM VERDAMPFEN VON FLÜSSIGKEITSTROPFEN

Zusammenfassung—Es wurde der Wärmeübergang an simulierte (poröse Kugel) Wasser- und Methanoltropfen in einem atmosphärischen Heißluftkanal gemessen. Die Experimente wurden durchgefürt für Re-Zahlen von 200 bis 2000, bei Anströmtemperaturen der Luft von 150°C bis 960°C und Geschwindigkeiten im Bereich von 2,1 m/s bis 11,4 m/s. Die experimentellen Werte können durch die Beziehung

$$Nu_f(1+B) = 2+0{,}6Re_M^{1/2}Pr_f^{1/3}$$

wiedergegeben werden, wobei die Stoffwerte für die Filmbedingungen eingesetzt sind, mit Ausnahme der Dichte für die *Re*-Zahl. In die *Re*-Zahl wird die Dichte der freien Strömung eingesetzt. Die Daten zeigen, daß die Wärmeübertragung bei höheren Temperaturen durch die Verdampfung um den Faktor $(1+B)$ verringert wird. Indirekte Verdampfung wirkt auf den Wärmeübergang infolge der Veränderung sowohl der Zusammensetzung als auch der Temperaturen des umgebenden Gasstroms.

ТЕПЛООБМЕН ПРИ ИСПАРЕНИИ КАПЕЛЬ ЖИДКОСТИ

Аннотация — Измерялся перенос тепла к модельным (пористая сфера) каплям воды и метилового спирта в вертикальной аэродинамической трубе с горячим воздухом при атмосферном давлении. Эксперименты проводились в диапазоне чисел Рейнольдса от 200 до 2000, температуре свободного потока воздуха от 150°C до 960°C и скорости от 2,1 м/сек до 11,4 м/сек. Экспериментальные данные наилучшим образом обобщаются соотношением

$$Nu_f(1+B) = 2+0{,}6Re_M^{1/2}Pr_f^{1/3},$$

в котором свойства определяются при пленочных условиях, за исключением плотности в числе Рейнольдса, которая представляет собой плотность свободного потока. Полученные данные свидетельствуют о том, что непосредственным эффектом испарения является снижение скорости переноса тепла в $(1+B)$ раз. Косвенное влияние испарения на скорость переноса тепла проявляется через изменение в составе и температуре окружающей газовой среды.